the Books of Daniel and Revelations; and finally, in 1913, *Our Own Religion in Ancient Persia.*

Another fruit of Dr. Mills's professional labors at Oxford, with which he was connected from 1898 on, is the *Dictionary of the Language of the Gāthas,* of which the first volume appeared in 1902 and the last in 1914, the fitting and final labor of a great and useful life.

CURRENT PERIODICALS.

Edward V. Huntington ("On Setting up a Definite Integral without the Use of Duhamel's Theorem," *American Mathematical Monthly,* Vol. XIV, 1917, pp. 271-275) makes a contribution of importance in the principles of the integral calculus. Consider the usual process of setting up an integral in the problem, say, of finding the total attraction P due to a thin rod of length $b-a$ at a point O in line with the rod and at distance a from the nearer end. Suppose the linear density of the rod to be any function, $f\ (x)$, which is known for all values of x from a to b. Also, suppose the attraction due to a particle to be proportional to F (x) times the mass of the particle. We actually proceed somewhat as follows. First, we think of the rod as divided into small elements, dx, where $dx=(b-a)/n$, and proceed to write down the attraction due to a typical element, say, from $x=x$ to $x=x+dx$. Thus, the mass of the element is seen to be $f(x)dx$, at least approximately—and the formula would be exact if the density throughout the element were the same as at its nearer end. Hence, the attraction at the point O due to the element $kF(x)f(x)dx$, at least approximately—and the formula would be exact if all the attracting material in the element were concentrated at its nearer end. In this k is a factor of proportionality. Having thus found the attraction due to a typical single element, at least approximately, we get the total attraction, P, due to all the elements, by integrating the last expression from a to b, "and in spite of the approximation used in setting up the integral, we feel assured that this final expression for P is exact."

Now, in many text-books, notably W. F. Osgood's *Calculus* of 1907 (revised edition 1909), the process of setting up an integral as the limit of a sum is held to require, for complete rigor, the use of "Duhamel's theorem." This theorem is as follows. If $\alpha_1, \alpha_2, \ldots \alpha_n$ is a set of positive infinitestimals such that

$$\lim [\alpha_1 + \alpha_2 + \ldots + \alpha_n] = \mathrm{A},$$

and if β_i, $\beta_2, \ldots \beta_n$ is a second set of positive infinitesimals such that each β differs from the corresponding α by an infinitesimal of higher order, so that $\lim [\beta_i/\alpha_i] = 1$; then

$$\lim [\beta_1 + \beta_2 + \ldots + \beta_n] = \mathrm{A}.$$

In these the limits are taken for n going to infinity. This theorem has exceptions, and examples of this falsity of the theorem are given, and it is to be noticed that although Osgood recognized this in a paper in 1903, he retained the incorrect form of Duhamel's theorem, without comment, in his text-book. Osgood gave his reasons for so doing in his article of 1903. If Duhamel's theorem is to be used at all, it must be taken in a modified form; and modified forms have been proposed by Osgood (1903), R. L. Moore (1912), and G. A. Bliss (1914). However in this article Huntington shows that the simple and uncritical process of integration regarded as a method of summation can be counted on to yield the correct result in the case, at least, when the functions $f(x)$ and $\mathrm{F}(x)$ are continuous. "It is not necessary to consider any questions of 'infinitesimal of higher order,' or any questions of 'uniformity'; the simple continuity of the two functions is sufficient." This theorem is stated and proved at some length.

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Louis C. Karpinski ("Algebraical Developments among the Egyptians and Babylonians," *American Mathematical Monthly*, Vol. XXIV, 1917, pp. 257-265) tries to show that "much of the material of our elementary algebra was long ago anticipated, to some extent, in the Orient. Similar anticipations of algebraical reasoning are indicated in the material, such as we have, which shows the progress of mathematics in ancient India and Greece.... In interpreting historical evidence one is constantly in danger of reading modern ideas into the text; on the other hand some writers in discussing Egyptian mathematics have been at great pains to discount the material which we have.... The Egyptians, even as early as 2000 B. C., attained a relatively high development in mathematics along analytical lines. This advance was made by the Egyptian priests who enjoyed that adequate leisure which is a primary essential for scientific advance. The assumption has frequently been made that the mathematics of the Egyptians was the product of their practical needs, this view being the result of a too serious regard for the statement of Herodotus that the Egyptians developed

geometry in order to redistribute the lands after the periodic overflow of the Nile. The assumption is absolutely refuted by a study of their mathematical achievements.... A just view of the mathematics involved must regard these points [practical application of mathematics] as applications and not at all as sources of the Egyptian mathematics. Fundamentally and universally mathematics is the achievement of thinking beings, occasioned by the mind and not by the body." Speaking of the Rhind papyrus, the author says that "the manual includes a number of problems in linear equations. The solution while essentially by the method of 'false position' is a definite and scientific procedure, leading to the correct value of the root of the equation. One of these first-degree equations is the following: '*Ahau* (heap, mass, unknown) and its seventh, it makes 19.' An arbitrary value, 7, is assumed as the root and the sum is found to be 8, instead of 19 as required; to obtain 19 from 8 the latter is doubled and multiplied by $\frac{1}{4}$ and $\frac{1}{8}$; the trial root 7 is also multiplied by 2, $\frac{1}{4}$, $\frac{1}{8}$, giving 16, $\frac{1}{2}$, $\frac{1}{8}$ as the value of the unknown; substitution of this value in the original equation follows, as a check, in accordance with the common procedure in Egyptian mathematics." After mentioning that symbols for the unknown, addition, and subtraction, and that simultaneous equations in two unknowns, leading to pure quadratics involving the Pythagorean triad $3^2+4^2=5^2$ are found in this and other ancient papyri, the author notices "that the Egyptian system of unit fractions, which persisted in Europe three thousand years after the times of the Ahmes manual, frequently gives a convenient method for actual computation." Also "the discussion in the Egyptian manual of arithmetical and geometrical progressions reveals an unexpected familiarity with rules which we now express by algebraical formulas, a familiarity which has not received adequate appreciation." There is mention of the weak point, which is apparently universal in Egyptian mathematics, in the discussion of the areas of triangles and trapezoids. "It is difficult to reconcile these crude approximations with the precision of measurements found in the construction of the pyramids and with the use of a method for drawing similar figures corresponding to the use of cross-section paper. The authorities are not in full agreement concerning the interpretation of the texts in question." In the section on algebraical ideas in Babylon, the author mentions the astronomical work of the Babylonians, their number symbols and decimal

and sexagesimal systems (the sexagesimal place system of recording numbers appears as early as 3000 B.C.), and the interest on the part of the Babylonians in arithmetical and geometrical series as early as 700 B.C. and in square and cubic numbers. "This brief survey of algebraical developments among the Egyptians and Babylonians shows that much of the material which was developed and extended by Greek mathematicians originated, both in methods and substance, with the scientists of the Orient." Φ

BOOK REVIEWS.

Religion and Science: a philosophical essay. By *John Theodore Merz.* Edinburgh and London, Blackwood and Sons, 1915. Pages, xi, 192. Price, 5s. net.

Clearly, this book belongs to a type. To be in love with emotion has been our affliction since Rousseau; to believe in belief is a form of the same malady. Mr. Merz knows Schleiermacher; he may or may not have read Maeterlinck, or Bergson, or Jean Jacques; but he cannot have escaped Goethe. As for romanticism in theology, we find one fundamental assumption: there is something called religion, independent of articulate creeds; there is the conviction that religion is so valuable that it must be "true"; and there is the prejudice that science is hostile to religion. Strong passions do not need explanation; but just as a man who is not very much in love excuses the follies which he has committed for the purpose of appearing passionate, so the philosophical Christian apologizes for the religion in which he would like to believe, and interprets the weakness of his opponents as evidence of his own strength. Maeterlinck exulted in the "*banqueroute de la science*" because it made religion again possible.

In this book the learned historian of European thought expounds three ideas: (1) Science deals only with an "external" world, which is a development of the world of common sense "with a still greater restriction of fundamental data" (p. 107) out of an earlier and larger reality. (2) Science describes and explains, its terms consist of "spatial data and their connections." Interpretation, i. e., the assignment (or the discovery?) of value and meaning, is reserved for religion. (3) Personality is that which is most real. The highest experience which we can have is the feeling of absolute dependence (Schleiermacher) which we trace to the influence of a higher power.

Mr. Merz decides, first, that the external world is a construction, that conceptual thought abstracts and selects. The products of this selection are subject and object, "an altered and fuller conception of reality," space, time, causality. These entities are carved out of a "primordial stream of thought" which apparently antedates thinking, which is a reality wider (though it is said to be less "full") than the external world. This internal possession is